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The effect of wall reflections on amplified spontaneous emission

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Abstract. The theory for amplified spontaneous emission is formulated when reflections from the bounding medium are important. The theoretical predictions are compared to previously reported, but quantitatively unexplained, results for the beam divergence of the 3·39 μm He–Ne system. Good agreement between theory and experiment is obtained.

1. Introduction

The phenomenon of amplified spontaneous emission (ASE) is now well understood both theoretically and experimentally (Allen and Peters 1973, to be referred to as I), although a full quantized field treatment is still lacking (Willis 1971). However, the effect of wall reflections on ASE is generally ignored and the geometry of the medium is used to define a solid angle into which spontaneous emission can be directed and so be emitted from the medium ends. In many approaches not even this aspect is accounted for and the useful spontaneous emission is assumed constant throughout the medium. Provided that the walls have not been rendered non-reflecting then to ignore such reflections is obviously a source of approximation in any analysis of laboratory ASE systems.

Wall reflections have been shown to be important in determining the properties of ASE by Andronova *et al* (1968) and Peters and Allen (1972, to be referred to as II), and for the coherence properties of light propagating along a passive tube of gas by Allen *et al* (1971). In laser systems wall reflections are usually ignored and the justification is that a low number transverse mode of laser radiation only occupies a small volume about the medium axis and hence has no contact with the walls. When considering lasers operating below threshold (Kimble and Mandel 1973) this does not seem to be a valid approximation to make in any physical situation. Masanko and Sviridov (1972) have considered a similar treatment to the one presented here for reflections suffered by spontaneous emission in a travelling-wave laser amplifier. However, they do not fully account for the spontaneous emission as a function of the spatial coordinates in the medium (Peters 1971).

2. Theory

In the previous approach to ASE (II) only that radiation which was spontaneously emitted into a certain solid angle ξ, defined at any point z in the medium by the medium end aperture, was considered. In principle, if reflection at the medium boundary can

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occur then emission into the whole 4π solid angle needs to be considered. However, for a large part of the 4π solid angle there will be no net gain as the loss suffered upon reflection will not be overcome by the gain experienced along the particular optical path.

To obtain a rough estimate of the angle over which spontaneous emission can be emitted and experience a net gain, consider a point A at position z in a two-dimensional medium of length L and width $2R$ as in figure 1. The angle over which spontaneous

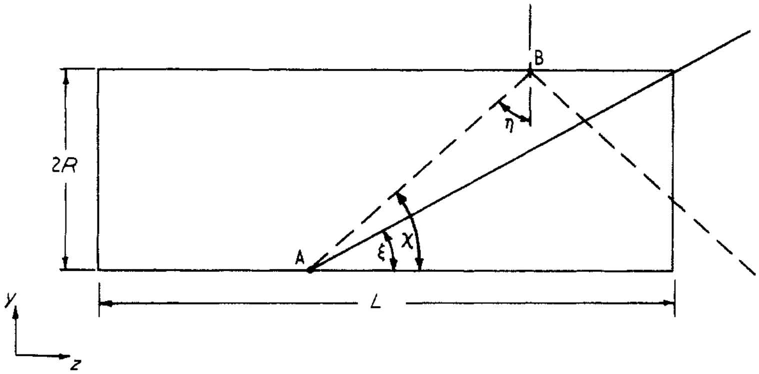

Figure 1. Side view geometry of the medium exhibiting ASE.

emission was considered to be important, in the previous theory, is indicated by ξ and the broken line indicates a possible path for radiation in the present approach. Let η_c be the maximum angle between the radiation path and the perpendicular to the medium boundary, such that the gain along the path AB just equals the losses suffered upon reflection. Ignoring any y dependence of the point A for the present, the appropriate gains through the medium, G_M, and upon reflection, G_R, at an angle η may be expressed as follows:

$$G_M = \exp(2R\alpha/\cos\eta) \qquad \text{and} \qquad G_R = R(\eta) \tag{1}$$

where α is the gain coefficient and $R(\eta)$ is the reflection coefficient at angle η. The reflection coefficient will depend upon the plane of polarization of the radiation (Longhurst 1967) and here unpolarized radiation is synthesised by taking an average of the two values, ie

$$R(\eta) = \frac{1}{2}\left(\frac{\sin^2(\eta-\eta')}{\sin^2(\eta+\eta')} + \frac{\tan^2(\eta-\eta')}{\tan^2(\eta+\eta')}\right) \tag{2}$$

where $\sin\eta' = (\sin\eta)/n$ and n is the refractive index of the bounding medium for the appropriate wavelength of the radiation. The derivation of this assumes that the bounding surfaces are perfectly flat, but any slight irregularities or thin films on the surface would affect the reflection coefficient in a real system. Hence η_c is defined by the relation $G_M G_R = 1$ or

$$2R\alpha/\cos\eta_c = -\ln R(\eta_c). \tag{3}$$

For the case of He–Ne emitting at 3·39 μm, in a pyrex tube, the following values are taken (I): $\alpha = 0{\cdot}020\ \text{cm}^{-1}$, $2R = 0{\cdot}25$ cm, and $n = 1{\cdot}45$. The angle η_c can then be easily computed and it transpires that $(\frac{1}{2}\pi - \eta_c) \simeq 2\times10^{-2}$ rad. The value taken here for α is the unsaturated value rather than the full form as discussed in I. Hence, in general, η_c will be both a function of L and z, as well as a function of y. However, consider a value of $L \simeq 200$ cm where saturation in the He–Ne system is not important, then the corresponding value of $\xi \sim 2R/L$ is about 10^{-3} rad. The calculation gives only an indication of the magnitude of η_c and merely shows that the amount of spontaneous emission that can act as a source for ASE can be very much underestimated if it is assumed that only that amount of emission into the angle ξ is important when reflections from the walls occur. The full transport equation will not be set up to consider the problem in more detail.

Consider a cylindrical volume of medium of length L and radius R. In cylindrical coordinates any point in the medium is defined by specifying (r, θ, z) and the direction of the emission from this point may be specified by (ϕ, χ) as in figure 2, where χ is defined

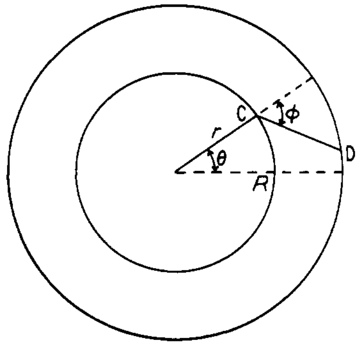

Figure 2. Cross section geometry of the medium exhibiting ASE.

as in figure 1 in a plane through CD parallel to the z axis. The number of reflections, M, suffered by radiation from such a point in this direction may be shown by simple geometric optics to be

$$M(r, z, \phi, \chi) = \left[\frac{[(L-z)\tan\chi/\cos\chi]+r'+r\cos\phi}{2r'}\right] \tag{4}$$

where

$$r' = [r^2\cos^2\phi+(R+r)(R-r)]^{1/2} \tag{5}$$

and the large square bracket denotes the maximum integer value less than or equal to the enclosed quantity. This expression is valid for any arbitrary point in the medium and fully accounts for the curvature of the bounding medium changing the plane through the direction of the emission and the medium axis after each reflection for all off-axis points. Note there is no θ dependence due to cylindrical symmetry.

The reflection coefficient is still given by equation (2) but in cylindrical coordinates the angle η has the form

$$\eta = \cos^{-1}[(r\cos\phi+r')\sin\chi/R] \tag{6}$$

where again the effect of the curvature of the bounding medium is fully accounted for. Although the loss occurs only at the medium boundary, it is more convenient to consider a distributed loss throughout the medium, eg the loss occurring at point B in figure 1 is assumed to be uniformly distributed along the path AB. So spontaneous emission experiences a loss characterized by a coefficient K_4 per unit length, where

$$K_4(r, z, \phi, \chi) = -M(r, z, \phi, \chi)\ln R(r, \phi, \chi)\cos\chi/(L-z). \tag{7}$$

The complete transport equation may now be expressed as

$$\frac{\partial I}{\partial z}(r, z, \phi, \chi) = (K_1(z) - K_4(r, z, \phi, \chi))I(r, z, \phi, \chi) + S(r, \chi) \tag{8}$$

where

$$K_1(z) = K_2/[1 + K_3(I(z) + J(z))],$$

K_2 is the small signal gain coefficient, K_3 is the saturation parameter and $I(z)$ and $J(z)$ are the total radiation intensities at position z in the medium in the + and − directions, respectively. The term $S(r, \chi)$ represents the spontaneous emission from an elemental volume, ie an annulus of radius r and width δr between the planes at z and $z+\delta z$, between two cones of half-apex angle χ and $\chi+\delta\chi$ and in the segment ϕ to $\phi+\delta\phi$. It may be shown that

$$S(r, \chi) = Kr \sin\chi\, \delta\chi\, \delta\phi\, \delta r \tag{9}$$

where K is a constant depending upon the total spontaneous emission per unit volume (I).

3. Comparison of theory with experiment

Consider the experimental data for the 3·39 μm He–Ne system previously reported (I). The constants of the system were deduced by solving equations (2) and (4)–(9) for $I(L)$ by the usual Runge–Kutta method of numerical evaluation and using the experimental data. The values of the constants are $K_2 = 0{\cdot}25 \times 10^{-1}$, $K_3 = 0{\cdot}18 \times 10^{-2}$, and $K = 0{\cdot}48 \times 10^5$.

The beam divergence can now be deduced by solving for $I(L, \chi)$, ie the total intensity between two cones of half-apex angle χ and $\chi+\delta\chi$. In this form it would be correct for application to the results of Andronova *et al* (1968), where an expanding diaphragm was used to determine the angular incremental change in intensity, but not for the 3·39 μm results (II). For this system a pinhole was tracked across the beam to determine the angular intensity distribution and hence the angular variation of the intensity per unit solid angle was measured, ie a quantity proportional to $I(\chi, L)/\sin\chi$.

Figure 3 shows the ASE beam divergence, evaluated according to various theories, as a function of medium length for an inversion density of 15·8 units and a medium bore of 2·5 mm. Curves A, B and C show, respectively, the simple $2R/L$ value, the ASE geometric value not accounting for reflections and the Koppelmann diffraction theory value, and the bars show the experimental results, all of which have been previously reported (II). Curve D indicates the theoretical values according to the theory presented in this paper. As can be seen, the agreement between theory and experiment is now very good, and the remaining discrepancy is probably due to the idealized situation assumed in the deduction of Fresnel's equation.

4. Conclusions

The theory for ASE when reflections from the walls of the bounding medium are important has been formulated. The experimental results for the beam divergence of the 3·39 μm He–Ne system are compared to predictions using this theory and the theory when reflections have not been accounted for. Better agreement is obtained using the former,

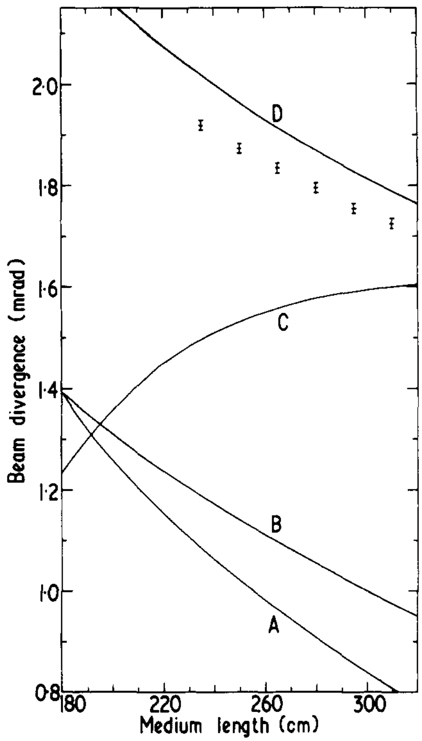

Figure 3. Beam divergence of the 3·39 μm He–Ne radiation as a function of medium length from a system with an inversion density 15·8 units and a diameter 2·5 mm, where the curves represent: A, simple $2R/L$ theory; B, ASE geometric approach; C, Koppelmann diffraction theory; D, ASE approach when reflections are accounted for; the bars show the experimental results.

although the idealized situation assumed in treating reflections leads to a value approximately 6% too high. The corresponding theory when reflections are ignored predicts a value approximately 40% too low. In the previous analysis (II) good agreement between theory and experiment was obtained in the pure neon system at 0·614 μm when reflections were not accounted for in the theoretical treatment. The reason for this was believed to be due to the reflections being diffuse here rather than specular as in the He–Ne system. It appears that to take account of reflections properly the exact nature of the reflecting surface needs to be known. However the two extreme cases, where reflections (i) can be ignored (diffuse reflections) and (ii) obey the Fresnel equations, can be treated adequately.

Acknowledgments

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